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Application of Monte-Carlo simulation towards a better understanding of Bayes' Theorem in Engineering Education

ABSTRACT

Bayes' Theorem (BT) is treated in probability theory and statistics. The BT shows how to change the probabilities a priori in view of new evidence, to obtain probabilities a posteriori. With the Bayesian interpretation of probability, the BT is expressed as the probability of an event (or the degree of belief in the occurrence of an event) should be changed, after considering evidence about the occurrence of that event. Bayesian inference is fundamental to Bayesian statistics. An example of practical application of this theorem in Health Systems is to consider the existence of false positives and false negatives in diagnoses. At the Academy, the theme of BT is exposed almost exclusively in its analytical form. With this article, the authors intend to contribute to clarify the logic behind this theorem, and get students better understanding of its important fields of application, using three methods: the classic analytical (Bayesian inference), the frequentist (frequency inference) and the numerical simulation of Monte-Carlo.

AIMS

We have noticed, in Probabilities and Statistics discipline, BT is most taught in an analytical way causing that a large part of students chooses to memorize the formulas rather than understand the inherent logic - since they do not get the whole understanding (deductive logic). This study aims to provide a computational tool to teach BT in engineering classes instead of a passive transfer of classic knowledge based on analytical methods. Besides, MS EXCEL is the only software tool needed which is widely available in most high educational facilities

This study aims to demonstrate how to use Monte-Carlo Simulation to get engineering students a better understanding of BT.

METHODS

To present a computational tool for applying Monte-Carlo Simulation, it is necessary to understand how active learning happens, exploring this subject. Additionally, it is important to understand how the probability and statistics discipline has been taught over time, to suggest a new and effective tool that help students to understand what BT all is about and what it is used for.
it was performed an online search about methodologies used in the discipline Probability and Statistics at Portuguese faculties of engineering, with special attention on BT. We have found eight high education institutions that provide the content of Probability and Statistics discipline on their websites. Most of the courses do not highlight methodologies used and others have focus on teaching how to use software like Software R and STATISTICA. No innovative tool was identified, what made us to believe that they use conventional methods and motivated us to develop a new approach using Monte-Carlo simulation, aiming to facilitate engineering students´ understanding of BT.

PROBABILITY AND STATISTICS

Probability and Statistics is an essential subject to teach engineering students. It has become a vital tool to engineers [1]. It is applied in so many areas such as manufacturing, development of food products, computer software, energy source and others [2]. Recently, research have shown interest to make mathematics closer to student´s experience. They recommend the use the real-life context in teaching, turning it in a prominent method to use [3].

MONTE CARLO SIMULATION

The case: The probability of any woman between 40 and 50 years of age to be sick from breast cancer is 0.008 (prior probability). If any woman, who is ill from this disease is submitted to a test, the probability that the test results positive is 0.9 (true positive). If the woman does not suffer from this disease, the probability that the test results positive is 0.07 (false positive). Under these circumstances, what is the probability that any woman, whose test has resulted positive, suffers from cancer? And in the case the test has resulted negative?".

The case described is programmed in EXCEL, named "Bayes test". It illustrates this idea and it can be downloaded here: https://www.dropbox.com/s/a8vl5enkbh46mhh/Bayes_test.xlsx?dl=0

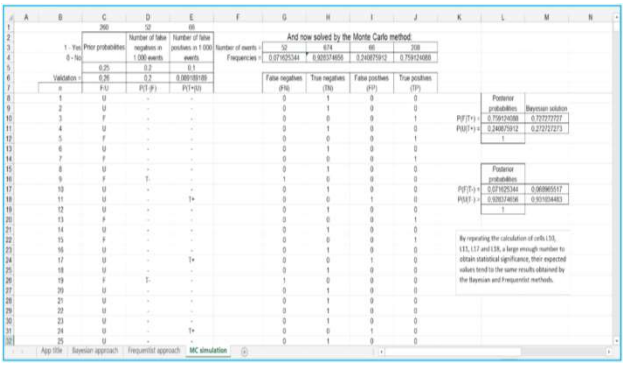

Figure 1. Screen of the "Simulation" sheet of the "Bayes" application

On the "Simulation" sheet (Figure 1), the results of n = 1,000 random tests are simulated, and the Monte-Carlo method calculates the posterior probabilities (or a posteriori).

DISCUSSION

We defend that students should concentrate on solving many application exercises. That is, instead of students spending time on the mechanics of resolution, they concentrate preferentially on the selection of the appropriate expression and on the interpretation of the results. The automatic method of calculation used also allows the performance of a quick sensitivity analysis of the results of each problem to different values of each of the input variables, helping to consolidate the concepts.
In conclusion, we strongly recommend the use of the numerical simulation technique for the construction of models that imitate the real world with just-enough precision and save time and money. While this technique is already trivial in many areas of Engineering and Management, it is not, however, in Statistics and hence our interest in the dissemination of this article.

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Rui Assis | p1028@ulusofona.pt
EIGeS-Research Centre in Industrial Engineering, Management and Sustainability, Lusófona University, Portugal

Pedro Carmona Marques | p4803@ulusofona.pt
EIGeS-Research Centre in Industrial Engineering, Management and Sustainability, Lusófona University, Portugal
ISEL-Polytechnic of Lisbon, Portugal

Raphaela Vidal | raphaela.vidal@ulusofona.pt
EIGeS-Research Centre in Industrial Engineering, Management and Sustainability, Lusófona University, Portugal